Int J Theor Phys (2010) 49: 2918–2928
DOI 10.1007/s10773-010-0484-1

Spin 0 and Spin 1/2 Particles in a Spherically Symmetric Static Gravity and a Coulomb Field

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Received: 19 April 2010 / Accepted: 1 September 2010 / Published online: 12 September 2010

Abstract A relativistic particle in an attractive Coulomb field as well as a static and spherically symmetric gravitational field is studied. The gravitational field is treated perturbatively and the energy levels are obtained for both spin 0 (Klein-Gordon) and spin 1/2 (Dirac) particles. The results are shown to coincide with each other as well as the result of the nonrelativistic (Schrödinger) equation in the nonrelativistic limit.

Keywords Relativistic particle · Gravitational field · Energy levels

1 Introduction

There has been a large collection of investigations on the systems in which both quantum mechanics and gravity are important. Some of these studies are on quantum fields in curved backgrounds, ([1] for example). Some others are on the behavior of first-quantized systems in curved backgrounds. Among these one can mention the simple study of a nonrelativistic particle in the presence of a constant gravity ([2], for example), the effect of gravity on neutrino oscillations ([3–5]), and also the study of relativistic quantum equations in a gravitational background. Chandrasekhar has studied the Dirac equation in a Kerr background [6], and others have extended this study in [7, 8] (for example). People have also studied the effect of spin. In [9] the possible difference between the Dirac Hamiltonians in a Schwarzschild background and a uniformly accelerated frame and the role of spin in this difference is addressed. In [10, 11], the solutions of the Klein-Gordon and Dirac equation in specific gravitational backgrounds have been studied and their differences have been addressed.

Here we are going to investigate the energy levels corresponding to bound states of charged particles in the field of a static charge in a curved static rotationally symmetric background. The system is studied perturbatively. The scheme of the paper is the following.

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In Sect. 2 the form of the Klein-Gordon and Dirac equation in a static rotationally symmetric background is reviewed. In Sect. 3, the energy levels corresponding to the Klein-Gordon and Dirac equation are obtained up to first order in the deviation of the metric from the metric of a flat space-time. In Sect. 4 the nonrelativistic limit is investigated. Section 5 is a discussion of the length scales involved, and Sect. 6 is devoted to the concluding remarks.

Throughout this text, the signature $(-+++)$ is used for the metric.

2 The Klein-Gordon and Dirac Equation in a Static Rotationally Symmetric Background

To fix notation, let us quickly review the form of Klein-Gordon and Dirac equations. The line element in suitable coordinates is

$$\begin{aligned} \mathrm{d}s^2 &= -c^2 A^2(r)\,\mathrm{d}t^2 + B^2(r)\,\mathrm{d}r^2 + r^2\,(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\phi^2), \\ &=: g_{\mu\nu}\,\mathrm{d}x^\mu\,\mathrm{d}x^\nu. \end{aligned} \tag{1}$$

The Klein-Gordon equation in the presence of a vector potential is

$$\begin{aligned} &\left[\frac{1}{\sqrt{|\det g|}}\left(-i\hbar c\frac{\partial}{\partial x^\mu} - U_\mu\right)\sqrt{|\det g|}g^{\mu\nu}\right. \\ &\quad \left.\times\left(-i\hbar c\frac{\partial}{\partial x^\nu} - U_\nu\right) - m^2 c^4\right]\psi_{\mathrm{KG}} = 0, \end{aligned} \tag{2}$$

where U is the vector potential times the particle's charge, and m is the particle's mass. For the line element (1), and if the only nonzero component of the vector potential is the zeroth (time) component, one arrives at

$$\left[-\left(i\hbar\frac{\partial}{\partial t} - V\right)^2 + c^2 A^2(-\hbar^2\nabla^2 + m^2 c^2)\right]\psi_{\mathrm{KG}} = 0, \tag{3}$$

where

$$\nabla^2 := \frac{1}{ABr^2}\frac{\partial}{\partial r}\frac{Ar^2}{B}\frac{\partial}{\partial r} + \frac{1}{r^2}\nabla^2_\Omega, \tag{4}$$

and

$$\nabla^2_\Omega := \frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\sin\theta\frac{\partial}{\partial\theta} + \frac{1}{\sin^2\theta}\frac{\partial^2}{\partial\phi^2}. \tag{5}$$

The Dirac equation is

$$\{\gamma^a[\hbar c(\partial_a + \Gamma_a) - iU_a] - mc^2\}\psi_{\mathrm{D}} = 0, \tag{6}$$

where the indices $a, b, \ldots$ denote tetrad indices, γ^a's are the Dirac matrices satisfying the Clifford algebra

$$\{\gamma^a, \gamma^b\} = 2\eta^{ab}, \tag{7}$$

in which

$$\eta^{ab} := \begin{cases} -c^{-2}, & a=b=0 \\ 1, & a=b\neq 0\ , \\ 0, & a\neq b \end{cases} \tag{8}$$

and Γ_a's are spin connections. These connections are found through

$$\Gamma_a := -\frac{1}{8}[\gamma_b, \gamma_c]\Gamma^c{}_a{}^b, \tag{9}$$

where $\Gamma^c{}_a{}^b$ is antisymmetric in c and b and satisfies

$$\mathrm{d}e^c + \Gamma^c{}_{ab} e^a \wedge e^b = 0, \tag{10}$$

in which e^a's form a tetrad basis:

$$e^a \cdot e^b = \eta^{ab} \tag{11}$$

choosing the tetrad

$$\begin{aligned} e^0 &:= A\,\mathrm{d}t, \\ e^1 &:= B\,\mathrm{d}r, \\ e^2 &:= r\,\mathrm{d}\theta, \\ e^3 &:= r\,\sin\theta\,\mathrm{d}\phi, \end{aligned} \tag{12}$$

corresponding to the line element (1), and assuming that the only nonzero component of the vector potential is the zeroth (time) component, one arrives at

$$\begin{aligned} \Bigg[-\left(i\hbar\frac{\partial}{\partial t} - V\right) - \frac{i\hbar c A}{B}\left(\frac{\partial}{\partial r} + \frac{1}{r} + \frac{1}{2A}\frac{\mathrm{d}A}{\mathrm{d}r}\right)\alpha^1 - \frac{i\hbar c A}{r}\alpha^1\beta\hat{K} \\ + mc^2 A\beta\Bigg]\psi_{\mathrm{D}} = 0, \end{aligned} \tag{13}$$

where

$$\hat{K} := \beta\left[\alpha^1\alpha^2\left(\frac{\partial}{\partial\theta} + \frac{1}{2}\cot\theta\right) + \alpha^1\alpha^3\frac{1}{\sin\theta}\frac{\partial}{\partial\phi}\right], \tag{14}$$

and

$$\begin{aligned} \beta &:= ic\gamma^0, \\ \alpha^j &:= c\gamma^0\gamma^j \end{aligned} \tag{15}$$

α^j's and β, of course satisfy

$$\begin{aligned} \{\alpha^i, \alpha^j\} &= 2\delta^{ij}, \\ \{\alpha^j, \beta\} &= 0, \\ \beta^2 &= 1. \end{aligned} \tag{16}$$

Finally, for the potential energy corresponding to a point charge in the field of a point charge at the origin, one has

$$\frac{\partial}{\partial x^j}(F^{j0}\sqrt{|\det g|}) = 0, \quad r \neq 0, \tag{17}$$

which (using spherical symmetry) results in

$$F^{rt} A B r^2 = \text{constant}, \tag{18}$$

or

$$F_{rt}(AB)^{-1} r^2 = \text{constant}, \tag{19}$$

where F is the field strength tensor. Using these, one arrives at the following equation for V:

$$\frac{\mathrm{d}V}{\mathrm{d}r} = \frac{kAB}{r^2}, \tag{20}$$

where k is a constant, actually minus the product of charges times the constant used in the Coulomb force expression.

3 Perturbative Calculation of the Energy Levels

Assume that the gravitational acceleration vanishes at $r = 0$. This means that the first derivatives of the metric vanish at the origin. Assuming that there is no singularity at $r = 0$ (so that there is no angle deficit), one arrives at

$$B(0) = 1. \tag{21}$$

By a suitable (constant) scaling of the time, one can also make

$$A(0) = 1. \tag{22}$$

So near $r = 0$ one can expand A and B like

$$\begin{aligned} A(r) &= 1 + \xi r^2, \\ B(r) &= 1 + \upsilon r^2, \end{aligned} \tag{23}$$

using these, up to first order in ξ and υ one arrives at

$$V = V_0 + k(\xi + \upsilon) r, \tag{24}$$

where

$$V_0 := -\frac{k}{r}. \tag{25}$$

3.1 The Klein-Gordon Equation

Using (4), one has (up to first order)

$$\nabla^2 = (1 - 2\upsilon r^2)\nabla_0^2 + \left[2\upsilon\nabla_\Omega^2 + 2(\xi - \upsilon) r \frac{\partial}{\partial r}\right], \tag{26}$$

where

$$\nabla_0^2 := \frac{1}{r^2}\frac{\partial}{\partial r} r^2 \frac{\partial}{\partial r} + \frac{1}{r^2}\nabla_\Omega^2. \tag{27}$$

Letting ψ_{KG} be a common eigenfunction of $i\hbar\partial/\partial t$ and ∇_Ω^2:

$$\begin{aligned} i\hbar\frac{\partial}{\partial t}\psi_{\mathrm{KG}} &= E\psi_{\mathrm{KG}}, \\ \nabla_\Omega^2\psi_{\mathrm{KG}} &= \lambda\psi_{\mathrm{KG}}, \end{aligned} \tag{28}$$

where

$$\lambda = -\ell(\ell + 1) \tag{29}$$

and ℓ is a nonnegative integer, one arrives (up to first order)

$$\begin{aligned} E^2\psi_{\mathrm{KG}} = \Bigg\{ & 2EV - V^2 + [1 + 2(\xi - \upsilon) r^2] c^2 (-\hbar^2\nabla_0^2 + m^2c^2) \\ & + 2m^2c^4\upsilon r^2 - \hbar^2 c^2 \left[2\upsilon\nabla_\Omega^2 + 2(\xi - \upsilon) r \frac{\partial}{\partial r}\right]\Bigg\}\psi_{\mathrm{KG}}. \end{aligned} \tag{30}$$

Multiplying both sides by ψ_{KG0} (the unperturbed eigenfunction corresponding to the unperturbed energy E_0), and normalizing ψ_{KG} like

$$\langle \psi_{\mathrm{KG0}}, \psi_{\mathrm{KG}} \rangle = 1, \tag{31}$$

one arrives at

$$\begin{aligned} E^2 = E_0^2 &+ \langle (2EV - V^2 - 2E_0V_0 + V_0^2) \rangle + \langle 2(\xi - \upsilon) r^2 c^2 (-\hbar^2\nabla_0^2 + m^2c^2) \rangle \\ &+ \left\langle \left\{ 2m^2c^4\upsilon r^2 - \hbar^2c^2 \left[+ 2\lambda\upsilon + 2(\xi - \upsilon) r \frac{\partial}{\partial r} \right] \right\} \right\rangle, \end{aligned} \tag{32}$$

where expectation values are calculated with the unperturbed eigenfunction. Finally, one has

$$\begin{aligned} (E_0 - \langle V_0 \rangle)\Delta E = {} & 2k^2\xi - \hbar^2c^2\lambda\upsilon + E_0 k(3\xi - \upsilon)\langle r \rangle \\ & + [E_0^2(\xi - \upsilon) + m^2c^4\upsilon]\langle r^2 \rangle - \hbar^2c^2(\xi - \upsilon)\left\langle r \frac{\partial}{\partial r} \right\rangle, \end{aligned} \tag{33}$$

where

$$\Delta E := E - E_0. \tag{34}$$

To obtain the expectation values, one notices that the unperturbed Klein-Gordon equation is like a Schrödinger equation with modified parameters:

$$\left(-\frac{\hbar^2}{2m}\nabla_0^2 + \frac{E_0 V_0}{mc^2} - \frac{V_0^2}{2mc^2}\right)\psi_0 = \frac{E_0^2 - m^2c^4}{2mc^2}\psi_0. \tag{35}$$

So defining

$$\begin{aligned} \epsilon_0 &:= \frac{E_0^2 - m^2c^4}{2mc^2}, \\ \tilde{k} &:= \frac{E_0 k}{mc^2}, \\ s(s+1) &:= \ell(\ell+1) - \frac{k^2}{\hbar^2c^2}, \end{aligned} \tag{36}$$

one arrives at

$$\epsilon_0 = -\frac{mc^2}{2}\frac{\tilde{k}^2}{\hbar^2c^2}\frac{1}{(n'+s+1)^2}, \tag{37}$$

where n' is a nonnegative integer. Then

$$E_0 = mc^2\left[1 + \frac{k^2}{\hbar^2c^2(n'+s+1)^2}\right]^{-1/2}. \tag{38}$$

One can then change (k, ℓ) to $(\tilde{k}, s)$ in the expectation values corresponding to the Schrödinger equation, to obtain the expectation values corresponding to the Klein-Gordon equation. One has (see for example [12])

$$\left\langle \frac{1}{r} \right\rangle = -\frac{2\epsilon_0}{\tilde{k}}, \tag{39}$$

$$\langle r \rangle = a\frac{3(n'+s+1)^2 - s(s+1)}{2}, \tag{40}$$

$$\langle r^2 \rangle = a^2\frac{(n'+s+1)^2[5(n'+s+1)^2 + 1 - 3s(s+1)]}{2}, \tag{41}$$

where

$$a := -\frac{\tilde{k}}{2(n'+s+1)^2\epsilon_0}. \tag{42}$$

Also, as $(1/r)(\partial/\partial r)r$ is antihermitian and r is hermitian, one has

$$\left\langle r\frac{1}{r}\frac{\partial}{\partial r}r + \frac{1}{r}\frac{\partial}{\partial r}rr \right\rangle = 0, \tag{43}$$

so that

$$\left\langle r\frac{\partial}{\partial r} \right\rangle = -\frac{3}{2}. \tag{44}$$

Putting all these together,

$$\Delta E = \frac{E_0\hbar^2c^2}{m^2c^4}\bigg[\frac{3}{2}(\xi-\upsilon)+2\frac{k^2}{\hbar^2c^2}\xi+\ell(\ell+1)\upsilon$$
$$+(3\xi-\upsilon)\frac{3(n'+s+1)^2-s(s+1)}{2}$$
$$+\left(\frac{E_0^2}{m^2c^4-E_0^2}\xi+\upsilon\right)\frac{5(n'+s+1)^2+1-3s(s+1)}{2}\bigg], \tag{45}$$

or

$$\frac{\Delta E_{\mathrm{KG}}}{mc^2} = f^2a_0^2\left[1+\frac{f^2}{(n'+s+1)^2}\right]^{-1/2}$$
$$\times\bigg\{\frac{3}{2}(\xi-\upsilon)+2f^2\xi+\ell(\ell+1)\upsilon$$
$$+(3\xi-\upsilon)\frac{3(n'+s+1)^2-s(s+1)}{2}$$
$$+\left[\frac{(n'+s+1)^2}{f^2}\xi+\upsilon\right]\frac{5(n'+s+1)^2+1-3s(s+1)}{2}\bigg\}, \tag{46}$$

where

$$f := \frac{k}{\hbar c},$$
$$a_0 := \frac{\hbar^2}{mk}. \tag{47}$$

3.2 The Dirac Equation

Using (13) and (24), one has (up to first order)

$$H_{\mathrm{D}} = [1+(\xi-\upsilon)]r^2]H_{\mathrm{D}0}+mc^2\upsilon r^2\beta+2k\xi r-i\hbar c\hat{K}\upsilon r\alpha^1\beta-i\hbar c\xi r\alpha^1, \tag{48}$$

where

$$H_{\mathrm{D}0} = V_0+mc^2\beta-i\hbar c\left(\frac{\partial}{\partial r}+\frac{1}{r}\right)\alpha^1-\frac{i\hbar c\hat{K}}{r}\alpha^1\beta. \tag{49}$$

It is then seen that

$$\Delta E = (\xi-\upsilon)E_0\langle r^2\rangle+mc^2\upsilon\langle r^2\beta\rangle+2k\xi\langle r\rangle-i\hbar cK\upsilon\langle r\alpha^1\beta\rangle-i\hbar c\xi\langle r\alpha^1\rangle, \tag{50}$$

where E_0 is the unperturbed energy, K is the eigenvalue corresponding to $\hat{K}$, and the expectation values are calculated using the unperturbed eigenvectors.

The expectation value of the commutator of anything with $H_{\mathrm{D}0}$ vanishes, in particular,

$$\langle[r^2, H_0]\rangle = 0, \tag{51}$$

which gives

$$\langle r\alpha^1\rangle = 0; \tag{52}$$

and

$$\langle [r^w \beta, H_0] \rangle = 0,$$
$$\langle [r^w \alpha^1, H_0] \rangle = 0,$$
$$\langle [r^w \alpha^1 \beta, H_0] \rangle = 0,$$
$$\left\langle \left[\left(\frac{\partial}{\partial r} + \frac{1}{r} \right) r^w, H_0 \right] \right\rangle = 0, \tag{53}$$

which give a set of recursive relations to obtain

$$X_w := \langle r^w \rangle,$$
$$Y_w := \langle r^w \beta \rangle,$$
$$Z_w := i\hbar c \langle r^w \alpha^1 \beta \rangle. \tag{54}$$

The relations are

$$2E_0 Y_w - 2mc^2 X_w + 2k Y_{w-1} + w Z_{w-1} = 0,$$
$$2mc^2 Z_w - \hbar^2 c^2 w X_{w-1} + 2\hbar^2 c^2 K Y_{w-1} = 0,$$
$$2E_0 Z_w + 2k Z_{w-1} + 2\hbar^2 c^2 K X_{w-1} - \hbar^2 c^2 w Y_{w-1} = 0,$$
$$-E_0(w+1) X_w + mc^2 (w+1) Y_w - kw X_{w-1} - K w Z_{w-1} = 0. \tag{55}$$

One also has

$$X_0 = 1,$$
$$Y_0 = \frac{1}{c^2} \frac{\partial E_0}{\partial m},$$
$$= \frac{E_0}{mc^2}, \tag{56}$$

where

$$E_0 = mc^2 \left(1 + \frac{k^2}{\hbar^2 c^2 \{ n'' + \sqrt{K^2 - [k^2/(\hbar^2 c^2)]} \}^2} \right)^{-1/2}, \tag{57}$$

and

$$|K| = j + \frac{1}{2}, \tag{58}$$

and $\hbar^2 j(j+1)$ is the eigenvalue of the total angular momentum squared. These can be found for example in [13].

Using (55) and (56), one arrives at

$$Z_1 = \frac{\hbar^2 c^2}{2mc^2} \left(1 - \frac{2K E_0}{mc^2} \right),$$
$$X_1 = \frac{k}{m^2 c^4 - E_0^2} \left(\frac{3E_0}{2} \right) - \frac{\hbar^2 c^2 K}{2mc^2 k} - \frac{\hbar^2 c^2 K^2 E_0}{2m^2 c^4 k},$$

$$E_0X_2 - mc^2Y_2 = -\frac{k^2E_0}{m^2c^4 - E_0^2} + \frac{\hbar^2c^2K^2E_0}{m^2c^4},$$

$$X_2 = \frac{k^2}{(m^2c^4 - E_0^2)^2}\left(2E_0^2 + \frac{m^2c^4}{2}\right) + \frac{\hbar^2c^2}{m^2c^4 - E_0^2}\left(\frac{1-K^2}{2} - \frac{3KE_0}{2mc^2} - \frac{K^2E_0^2}{m^2c^4}\right), \tag{59}$$

where

$$\Delta E = \xi E_0X_2 - \upsilon(E_0X_2 - mc^2Y_2) + 2k\xi X_1 - K\upsilon Z_1. \tag{60}$$

So one has

$$\frac{\Delta E_{\mathrm{D}}}{mc^2} = \xi a_0^2\left[f^4\frac{7\zeta^3 - 2\zeta}{2(\zeta^2-1)^2} - f^2\frac{2K\zeta^2 + (3K^2-1)\zeta + K}{2(\zeta^2-1)}\right] + \upsilon a_0^2\left(f^4\frac{\zeta}{\zeta^2-1} - f^2\frac{K}{2}\right), \tag{61}$$

where

$$\zeta := \left[1 + \frac{f^2}{(n'' + \sqrt{K^2 - f^2})^2}\right]^{1/2}. \tag{62}$$

4 The Nonrelativistic Limit

To obtain the nonrelativistic limit of the expressions (46) and (61), one should take into account that ξ and υ themselves are proportional to c^{-2}. So expansion of the left hand sides of (46) and (61) up to order f^2 gives the nonrelativistic value of ΔE and its first relativistic correction. Defining the principal quantum number through

$$n := n' + \ell + 1, \tag{63}$$

in the Klein-Gordon equation and

$$n := n'' + |K|, \tag{64}$$

in the Dirac equation, one arrives at

$$\Delta E_{\mathrm{KG}} = m\left\{(\xi c^2a_0^2)\frac{n^2[5n^2 - 3\ell(\ell+1) + 1]}{2} + (\xi c^2a_0^2)f^2\left[-\frac{10n^3+n}{2l+1} + \frac{19n^2+5}{4} - \frac{3\ell(\ell+1)}{4} + \frac{3n\ell(\ell+1)}{2\ell+1}\right] + (\upsilon c^2a_0^2)f^2(n^2-1)\right\} + o(f^2), \tag{65}$$

and

$$\Delta E_{\mathrm{D}} = m\bigg\{(\xi c^2 a_0^2)\frac{n^2[5n^2 - 3K(K+1)+1]}{2} + (\xi c^2 a_0^2) f^2\bigg[-\frac{10n^3+n}{2j+1} + \frac{19n^2}{4} + \frac{3nK}{2j+1} + \frac{(2j+1)^2+4}{16} - K(K+1) + \frac{3n(2j+1)}{4}\bigg] + (\upsilon c^2 a_0^2) f^2\bigg[n^2 - \frac{K}{2}\bigg]\bigg\} + o(f^2), \tag{66}$$

where (58) has been used. In particular it is seen that in the limit $c \to \infty$, the perturbed energy is the same for the Klein-Gordon and Dirac equation:

$$\Delta E_{\mathrm{NR}} = m(\xi c^2 a_0^2)\frac{n^2[5n^2 - 3\ell(\ell+1)+1]}{2}, \tag{67}$$

where

$$K^2 + \beta K = \frac{1}{\hbar^2}\mathbf{L}\cdot\mathbf{L} \tag{68}$$

has been used (in which $\mathbf{L}$ is the orbital angular momentum), and the fact that

$$\lim_{c\to\infty} \langle \beta - 1\rangle = 0. \tag{69}$$

Equation (67) is equivalent to

$$\Delta E_{\mathrm{NR}} = m(\xi c^2)\langle r^2\rangle_{\mathrm{NR}}, \tag{70}$$

which is nothing but the expectation value of the gravitational potential, as in the nonrelativistic limit the only relevant term in the metric is the potential which is related to A through

$$\bigg[1 + \frac{\phi_{\mathrm{gr}}}{c^2}\bigg]^2 = A^2. \tag{71}$$

5 Length Scales

It is seen from previous sections, (65) and (66), that the shift in the energy levels up to leading order in f is of order $mc^2\xi a_0^2$ and $mc^2\upsilon a_0^2 f^2$. Assuming that f is small and ξ and υ are of the same order L^{-2}, where L^{-2} is of the order of the curvature corresponding to the metric, it is seen that

$$\Delta E \sim mc^2\bigg(\frac{a_0}{L}\bigg)^2, \tag{72}$$

or

$$\Delta E \sim (mc^2 f^2)\bigg(\frac{a_0}{fL}\bigg)^2. \tag{73}$$

In order that the (23) be valid, ξa_0^2 and υa_0^2 should be much less than unit, that is

$$a_0 \ll L. \tag{74}$$

In order that treating the gravitational field as a perturbation to the Coulomb field be valid, the energy shift due to gravity should be much less than the unperturbed energy levels (minus the rest energy of the particle of course). This reads

$$a_0 \ll f L. \tag{75}$$

Comparing (74) and (75), it is seen that for small Coulomb couplings (small values of f), it is (75) which is more restrictive. So the overall criterion is (75). If this is satisfied, then the transformation of the metric to something like a Schwarzschild solution occurs at a length scale larger than the size of the system (a_0) divided by f, which is much larger than the size of the system. So the form (23) is valid for the system.

For a hydrogen atom, this means that the length scale corresponding to the gravitational field should be much larger than a hundred Bohr radiuses. That is, L should be much larger than 10^{-8} m.

6 Concluding Remarks

A static spherically symmetric gravitational field which vanishes at the origin is determined by two constants near the origin. A particle in a Coulomb field as well as such a gravitational field was studied. If the gravitational field is not very strong so that its corresponding inverse curvature is much larger than the size of the bounded system squared, it is plausible to treat the gravitational field perturbatively. This was done and the perturbed energy levels were obtained for spin 0 and spin $1/2$ particles. The results were further expanded in terms of the inverse of the speed of light, to deduce the nonrelativistic parts as well as the first relativistic corrections. It was seen, as expected, that in the nonrelativistic limit the shift of energy levels due to gravitation for the spin 0 and spin $1/2$ particles coincide, and both coincide with the result of the Schrödinger equation. In fact the effect of the spin on the energy shift vanishes in this limit. In this limit also only one of the parameters of the background metric enters the energy shift, the parameter which determines the time-time component of the metric. This too, is expected, as in the nonrelativistic limit only this parameter (which determines the nonrelativistic gravitational potential) enters the equation of motion of the particle. Of course the relativistic corrections contain the other parameter as well as spin-dependent terms.

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